



Thermodynamic measurement and analysis of dual-temperature thermoacoustic oscillations for energy harvesting application



Dan Zhao^{a,*}, Chenzhen Ji^a, Shihuai Li^a, Junwei Li^b

^a Division of Aerospace Engineering, School of Mechanical and Aerospace Engineering, College of Engineering, Nanyang Technological University, N3-02c-72, 50 Nanyang Avenue, Singapore 639798, Singapore

^b School of Aerospace Engineering, Beijing Institute of Technology, Beijing 100081, China

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ABSTRACT

The present work considers energy harvesting by implementing both thermo- and piezo-electric power generation modules on a bifurcating tube, which produces dual-temperature thermoacoustic oscillations. The present system distinguished from the conventional standing-wave one does not involve heat exchangers and uses two different energy conversion processes to produce electricity. To measure and analyze the sound waves generated, an infrared thermal imaging camera, hot wire anemometry, and two arrays of K-type thermocouples and microphones are employed. It is found that the total electric power is approximately 5.71 mW, of which the piezo module produces about 0.21 mW. It is about 61% more than that generated by a similar conduction-driven thermo-acoustic-piezo harvester. In order to gain insight on the heat-driven acoustic oscillations and to simulate the experiment, thermodynamic laws are used to develop a nonlinear thermoacoustic model. Comparison is then made between the numerical and experimental results. Good agreement is obtained in terms of frequency and sound pressure level. Finally, Rayleigh index is examined to characterize the conversion between thermal and sound energy. In addition, energy redistribution between different thermoacoustic modes is estimated. It is found that lower frequency thermoacoustic oscillations are easier to trigger.

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1. Introduction

Fossil fuel exhaustion and greenhouse gas emission are increasingly urgent issues. However, waste heat generated by burning fossil fuel is released to the atmosphere in industry, which is harmful to the environment and energy wasting. To recover waste heat, there is a need for energy conversion techniques. Thermoacoustics [1–6] is a research field which deals with the conversion from thermal to acoustical energy [7,8]. For refrigeration, heat should be transferred into the working gas of a thermoacoustic system during the expansion [7], and dumped from the gas during compression. A thermoacoustic heat engine (also known as prime mover) operates on a thermodynamic cycle in the direction opposite to that of the refrigeration cycle. Theories and models have been developed and reviewed for thermoacoustic phenomenon and devices [2,5,7].

Thermoacoustic engines are more attractive than the traditional Stirling engines [9] involving pistons, due to the fact that they lack moving parts, require little maintenance and last long. However,

these engines [10,11] provide lower efficiency, little reliability, and high fabrication cost. For example, the traveling-wave engines [11] have high efficiency and high reliability, but have high fabrication costs due to the presence of regenerator and heat exchangers. The standing-wave engines are reliable, and cheaper to fabricate; but they are inefficient. Typically, a thermoacoustic engine is one of the thermofluidic oscillators in which thermodynamic oscillations are generated and sustained by external temperature differences [12] achieved by heat exchangers that play critical roles in producing such oscillations. However, implementing such heat exchangers causes not only additional energy consumption but also increased manufacturing cost.

Thermoacoustics systems involve the energy conversion from heat to sound or other form of mechanical work [13–15]. Optimization of the energy conversion process was studied by several researchers [6,15]. Various forms of input energy have been tested such as solar power [13] and concentrated laser pair [16]. A miniature thermoacoustic engines were developed [17]. Recently, a thermo-acoustic-piezo system was designed and tested to achieve energy conversion from heat-to-sound-to-electricity [18]. It was shown that the output power was very low (approximately 0.128 mW) and two heat exchangers were used.

* Corresponding author.

E-mail address: zhaodan@ntu.edu.sg (D. Zhao).

In this work, a thermal energy harvesting system is designed and experimentally tested. For this, a cylindrical tube splitting into two bifurcating daughter branches is designed. As a heat source is placed inside the mother tube, it provides a mechanism to produce both ‘hot’ and ‘cold’ thermoacoustic oscillations (also known as Rijke–Zhao oscillations). To achieve energy harvesting in the Y-shaped tube, a piezoelectric diaphragm is implemented to the bifurcating branch with ‘cold’ oscillations. And a thermoelectric power generation module is attached to the end of the other bifurcating branch with ‘hot’ oscillations. The experimental setup is described in Section 2. Thermodynamic measurements are then conducted. This involves measuring the temperature and monitoring the sound fields, as described in Sect. 3.1. The performances of the piezo- and thermoelectric modules are then measure and compared, as described in Section 3.2. Finally, in Section 4, thermodynamic laws are used to develop a nonlinear thermoacoustic model to gain insight on the heat-to-sound energy conversion and to simulate the experiment. Comparison is then made between the numerical and experimental results. In addition, Rayleigh index and sound energy redistribution are examined to characterize the energy conversion, and the energy redistributed between different eigenmodes.

2. Experimental setup

A non-axisymmetric bifurcating tube with a thermo- and piezo-electric diaphragm attached is designed and tested to demonstrate the possibility of harvesting thermal energy by using two different energy conversion processes, as show in Fig. 1. The angle between the bifurcating branches is 2α and $0^\circ < \alpha < 90^\circ$. As a heat source (flame or solar heater et al.) is placed inside the mother tube of length 0.3 m, it provides a mechanism to produce dual-temperature thermoacoustic oscillations [19,20]. It is worth noting that the oscillations generated in the present system are convection-driven ones, which are different from the conventional conduction-driven standing- or traveling-wave prime movers [1,7]. T-shaped junctions protruding from the bifurcating branches are designed for placing thermocouples, microphones or hot wire anemometry. Before the thermodynamic measurements are conducted, the microphones are calibrated by using a piston phone (124 dB at 250 Hz). The measurement error is about ± 0.1 dB. The hot wire anemometry is calibrated by using a Pitot-static tube. The thermocouples are connected to a Pratos K-type digital

thermometer (DE305) and compared with the measurement from our National Instrument PXIE4353 thermocouple module.

The thermoelectric module as shown in Fig. 2(a) and (b) is a $40(W) \times 40(L) \times 3.8(T)$ mm square diaphragm (GM250-127-14-16). Its technical data is shown in the top row of Table 1. The piezoelectric module is a circular diaphragm (Piezo Systems Inc. T216-A4NO-573X) made of Lead–Zirconate–Titanate, as shown in Fig. 2(c) and (d). It has thickness of 0.41 mm at resonant frequency of 290 Hz. It's technical data is shown in the bottom row of Table 1.

3. Experimental results

3.1. Thermodynamic measurements

To measure the temperature distribution in the bifurcating branches, an infrared thermal imaging camera is used. It captures the quartz tube surface temperature spectrum. However, it can characterize the ‘hot’ status of the oscillating flow inside each branch. The camera is set to operating range of 0 to 500 °C. Fig. 3(a) shows the measured temperature contour of the present setup in comparison with the temperature measurement of a conventional Rijke-type thermoacoustic system as shown in Fig. 3(b). It can be seen that the present system is associated with two different temperature ‘exhaust’ flows. The surface temperature of the upper short branch is around 26 °C, indicating that the air flow inside the branch is very close to room temperature, while the other upper branch is associated with ‘hotter’ air flow. However, the conventional Rijke-type system involves with only ‘hot’ exhaust oscillating flow, which limits the application of a piezoelectric generator due to the high temperature. Thus our present thermoacoustic system is unique in terms of producing dual-temperature oscillations, and providing a platform to apply energy harvesters with both thermoelectric and piezoelectric energy conversion processes.

To validate the temperature measurement, two arrays of K-type thermocouples are implemented on the bifurcating tube: one array with 4 thermocouples connected to the longer upper branch of length 0.9 m, the other with 3 thermocouples attached to the shorter branch of length 0.5 m. The measured temperatures are summarized in Fig. 4. It can be seen that one branch is with a ‘hot’ flow, while the other one is at ambient temperature. These measurements are consistent with the infrared camera measurement. The measurement difference between the infrared camera and the thermocouples is approximately ± 2 °C.

The presence of the thermoacoustic oscillations can be confirmed by measuring the pressure fluctuations along the bifurcating branches, since they cause the air pressure fluctuating. For this, two arrays of microphones (B&K Type 4954) are implemented and placed side by side to the thermocouples. The measurements are shown in Fig. 5(a) and (b). It can be seen that there are intensive pressure oscillations in both bifurcating branches, and the maximum sound pressure level is about 137 dB. The dominant mode of the thermoacoustic oscillation is at around 180 Hz. And it has several harmonics (see Fig. 5(a) and (b)), which indicates the nonlinearity of the system. This frequency corresponds to the fundamental mode with a wavelength twice of the total length of the bottom stem and the upper longer branch, while a conventional Rijke-type system with total length of 0.8 m is associated with around 230 Hz oscillations, as shown in Fig. 5(c). The ‘oscillating’ nature of thermoacoustics in the bifurcating branches is further confirmed via direct measurement by using a HWA (hot wire anemometers) near the exit (about 0.5 cm) of the ‘cold’ branch, as shown in Fig. 5(d). This confirms that the present setup is a reliable system to produce dual-temperature oscillations.

It is worth noting that the angle α between the two bifurcating branches plays an important role in producing dual-temperature

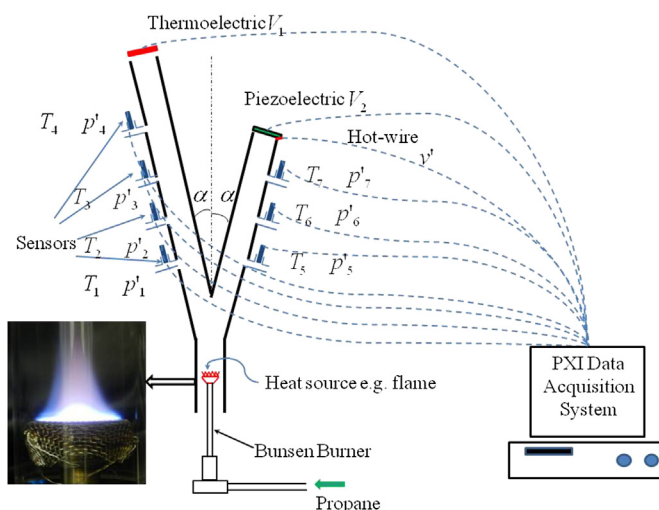


Fig. 1. Experimental setup of dual-temperature thermoacoustic system with a piezo-electric and thermoelectric power generator attached.

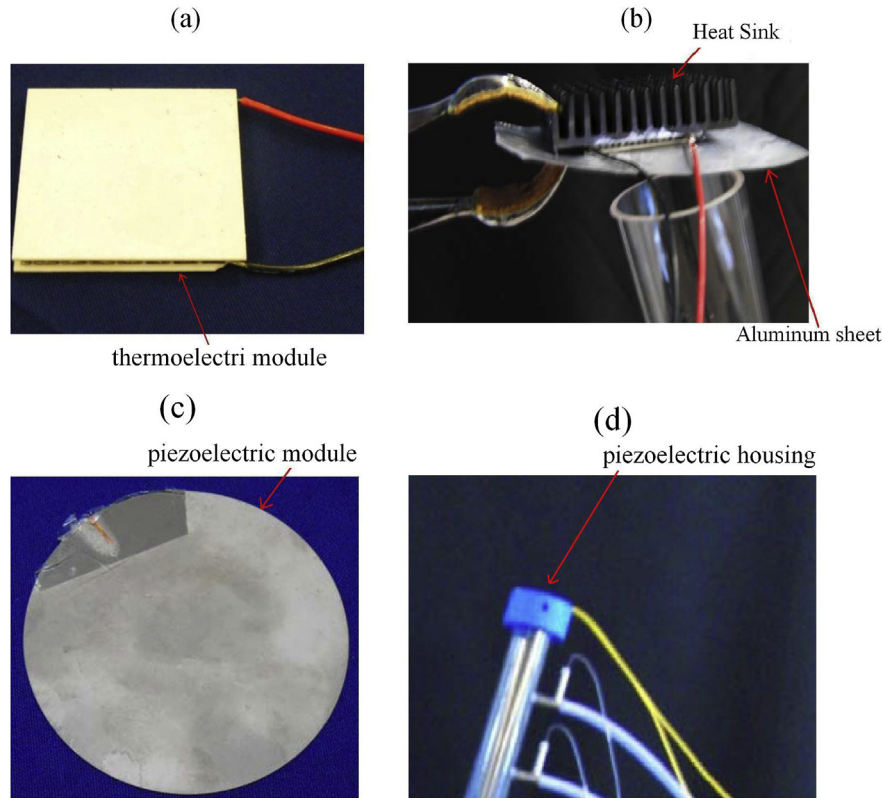


Fig. 2. Photo of thermo- and piezo-electric power generators and their implementation.

oscillations. To confirm it, temperature contour measurement of another bifurcating tube with $\alpha = 90^\circ$ is conducted, as shown in Fig. 6(a). It can be seen that both ‘hot’ oscillating flows are produced. The oscillatory nature of the ‘exhaust’ flow can be observed in the measurement of the pressure fluctuation phase diagram along the left and right branches, as shown in Fig. 6(b). The ‘hot’ oscillations produced are due to the buoyancy generated, which is related to the height of the bifurcating branch. If the buoyancy force is characterized by reduced gravity g' [21], then it can be shown as

$$g' = gH \frac{\nabla T}{T} \quad (1)$$

where H denotes the height of the upper branch open end with respect to the bottom open end. When $\alpha = 90^\circ$, $g'_L = g'_R$. This gives rise to both bifurcating branches are involved with ‘hot’ oscillating flows. In order to study the effect of the angle α at different values on producing dual-temperature oscillations, further investigations are needed to determine the optimum geometric configuration.

3.2. Electrical power generation measurement

The oscillations consisting of both ‘hot’ and ‘cold’ pressure fluctuations might have potential application in industry, such as energy efficient drying for small-size particles such as soya beans or

coffee beans [6]. However, in the present work, we demonstrate another potential of using such oscillations for energy harvesting. This is achieved by implementing a thermoelectric generator in the ‘hot’ bifurcating branch and a piezoelectric generator in the branch with ‘cold’ oscillations, as shown in Fig. 1. Thus the thermal energy harvested is maximized by simultaneously using two different energy conversion processes, which is different from the conventional standing-wave thermo-acoustic-piezo system [18].

Fig. 7 shows the electric voltage output and the power generated from both the TE (thermoelectric) and PE (piezoelectric) modules. It can be seen that the total power output from PE and TE is about 5.71 mW, of which about 5.5 mW is produced by the TE. The PE generates approximately 0.21 mW electricity, which is much lower than TE. However, when it is compared with the electrical power generated from a similar PE implemented on a conduction-driven thermoacoustic system [18], our PE generates 61% more power, as shown in Fig. 7(d). The PE performance is related to the oscillation frequency. In order to maximize its electricity output, it is tuned to ensure that its resonance frequency is closed to the oscillation frequency 180 Hz by adding an aluminum disk-plate weighting 2.85 g at the center of the diaphragm. Similar tuning was conducted by Smoker et al. [18]. Note that the electrical power output E_L is calculated by:

$$E_L = V_{\text{rms}}^2 / R_L = V^2 / 2R_L \quad (2)$$

where $V_{\text{rms}} = V/\sqrt{2}$ denotes the root mean square of the output voltage V .

The energy conversion efficiency from thermal to electric power is also measured. Fig. 8 shows the overall efficiency of converting input heat power into output electrical power via the thermoelectric η_e and piezoelectric modules η_a . It can be seen that the maximum η_e is about 0.0013%, when the loading resistance is approximately 7 Ω . The overall energy conversion efficiency of the

Table 1
Parameters of thermo- and piezo-electric power generators.

Couples	V_{oc} (V)	I_{max} (A)	P_{max} (W)	T_{max} (C)	R_{in} (Ω)
127	9.4	1.09	5.10	250	7.9
Weight (g)	Stiffness (N/m)	Capacitance (nF)	Blocked force (N)	Diameter (mm)	Free deflection (μm)
9.8	5×10^3	107	± 2.4	63.5	± 476

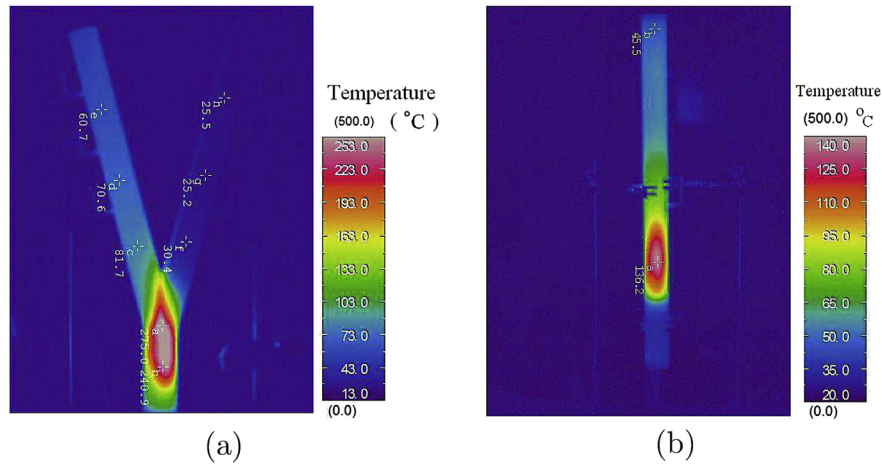


Fig. 3. Temperature contour measurement by using infrared thermal imaging camera (a) dual-temperature thermoacoustic system (b) conventional Rijke-type system.

TE module η_e is much larger than that of PE η_a . However, when the present system is compared with the thermo-acoustic-piezo system as reported by Smoker et al. [18], the maximum η_a is found to be increased by 60%, as showed in Fig. 8(b).

In order to gain insight on the heat-to-sound-to-electricity conversion, the conversion efficiency η_c from acoustical to electrical energy is measured. η_c is defined as $\eta_c = E_L/p \cdot \mathcal{V}$, where p and $\mathcal{V}$ denote the pressure and volume velocity at the piezo-diaphragm position. It can be seen that the piezoelectric diaphragm gives rise to the maximum energy conversion efficiency of approximately 22% when the load resistance is 5070 Ω . It is much greater than the maximum acoustical energy conversion efficiency of 9.5% obtained by Smoker et al. [18]. Furthermore, our system uses two different energy conversion processes. And it involves no heat exchangers and stack, which are critical components in conventional thermo-acoustic-piezo system [18]. Note that further investigations are needed to minimize the size of such dual-temperature thermoacoustic system and to maximize its performance so that it can be practically applied.

4. Theory of heat-driven Rijke–Zhao oscillations

4.1. Thermodynamic analysis

The pressure of the gas mixtures used in the present setup is always low enough to justify the assumption of the perfect gas laws. Thus if p denotes the instantaneous pressure, T the temperature and ρ the density,

$$p = \rho T \frac{\mathcal{R}_u}{\mathcal{M}_W} = \frac{\rho c^2}{\gamma} \quad (3)$$

where c is sound speed, $\gamma = 1.4$ is the ratio of specific heats, $\mathcal{R}_u$ is the universal gas constant ($=8315$ J/kmol-K), $\mathcal{M}_W$ is the molar mass of the gas mixture. Since the fuel burned is LPG (liquefied petroleum gas) with main component of C_3H_8 [22], the complete chemical reaction in single step Arrhenius kinetics is of the form



Then the rate of the reaction is given as

$$\frac{d[C_3H_8]}{dt} = -\mathcal{A}_f e^{-\mathcal{E}_a/\mathcal{R}_u T} [C_3H_8]^b [O_2]^c \quad (5)$$

where $\mathcal{A}_f$ is the Arrhenius constant describing the collision frequency of molecules. $\mathcal{E}_a$ is the activation energy. $[\cdot]$ denotes the concentrations of reactant species. b and c are the reaction rate

parameters, which are set to 0.1 and 1.65 respectively [23]. If $\mathcal{H}$ denotes the energy release per gram of the gas mixtures by the complete reaction, the rate at which heat is generated in the gas per gram is given by

$$Q(T, t) = -\frac{\mathcal{H}}{[C_3H_8]_{t=0}} \frac{d[C_3H_8]}{dt} = \bar{Q}(T) + Q'(T, t) \quad (6)$$

where the instantaneous heat release $Q(T, t)$ consists of a mean value $\bar{Q}(T)$ and a perturbation part $Q'(T, t)$.

If we denote the internal energy of the gas mixture by E , then

$$E = \frac{c_v p}{\mathcal{R} \rho} = \frac{1}{\gamma - 1} \frac{p}{\rho} \quad (7)$$

where c_v is the constant-volume specific heat and $\mathcal{R} = \mathcal{R}_u/\mathcal{M}_W$. The first law of thermodynamics considering energy conservation can be described by

$$\frac{dE}{dt} = \frac{\partial Q(T, t)}{\partial t} - p \frac{d(1/\rho)}{dt} \quad (8)$$

The convection flow in the present system has negligible Mach number $\bar{M}_a$ [24,25], so that the flow properties in a region of the gas

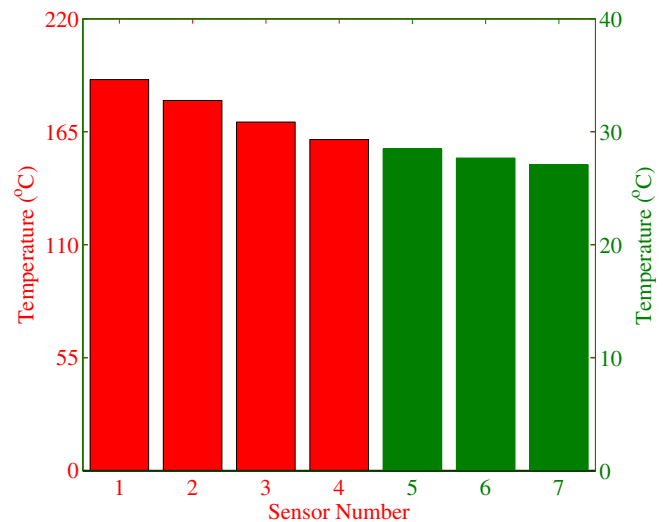


Fig. 4. Temperature measurements by using two arrays of K-type thermocouples along the bifurcating branches.

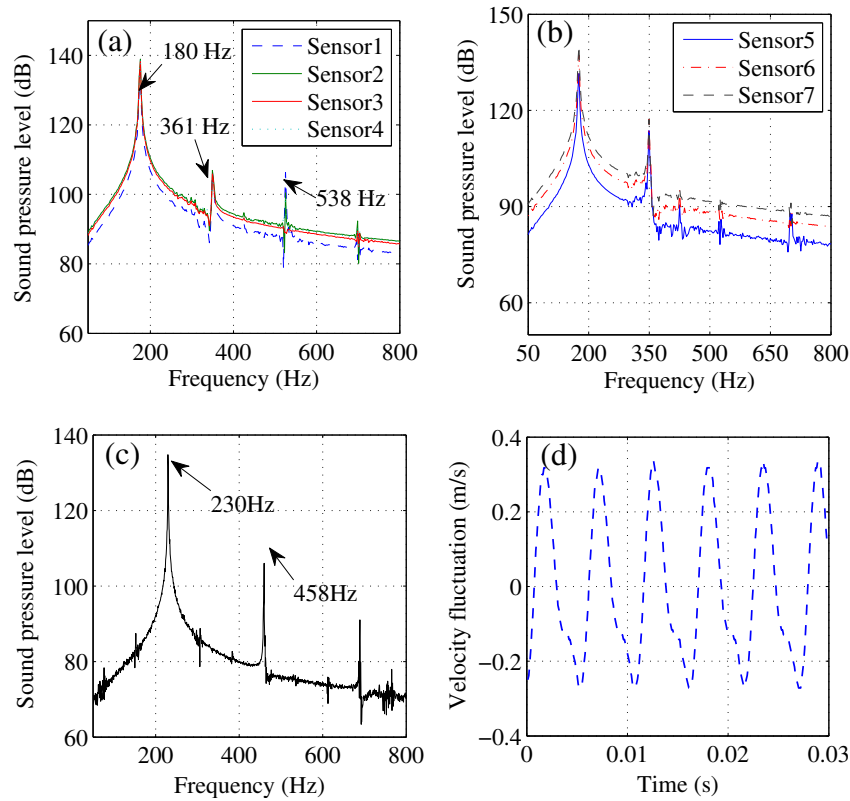


Fig. 5. Measured thermoacoustic oscillations: (a) frequency spectra along the upper long branch, (b) frequency spectra along the upper short branch, (c) frequency spectra along the conventional Rijke-type system with total length of 0.8 m and (d) velocity fluctuation near the upper short branch open end.

mixture where the mean pressure, temperature, density and velocity are $\bar{p}$, $\bar{T}$, $\bar{\rho}$ and $\bar{u}$ respectively can be expanded as

$$\frac{p}{\bar{p}} = 1 + \bar{M}_a \sigma_1 + \mathcal{O}(\bar{M}_a^2 \sigma_2) + \dots, \quad \frac{T}{\bar{T}} = 1 + \bar{M}_a \tau_1 + \mathcal{O}(\bar{M}_a^2 \tau_2) + \dots \quad (9)$$

$$\frac{\rho}{\bar{\rho}} = 1 + \bar{M}_a \chi_1 + \mathcal{O}(\bar{M}_a^2 \chi_2) + \dots, \quad \frac{u}{\bar{u}} = 1 + \bar{M}_a \mu_1 + \mathcal{O}(\bar{M}_a^2 \mu_2) + \dots \quad (10)$$

Substituting the linearized form of Eqs. (9) and (10) by neglecting the second and higher order terms into Eq. (8) leads to

$$\frac{d}{dt} \ln \left\{ (1 + \bar{M}_a \tau_1) (1 + \bar{M}_a \chi_1)^{1-\gamma} \right\} = \frac{\gamma - 1}{(1 + \bar{M}_a \tau_1) \mathcal{R} \bar{T}} \frac{\partial Q(T, t)}{\partial t} \quad (11)$$

Application of Taylor series expansion on Eq. (11)¹ and neglecting second order terms gives rise to the linearized form of Eq. (11) as follow

$$\frac{\gamma - 1}{\mathcal{R} \bar{T}} \frac{\partial Q(T, t)}{\partial t} = \frac{\partial}{\partial t} (\bar{M}_a \tau_1 + (1 - \gamma) \bar{M}_a \chi_1) \quad (12)$$

The motion of the perfect gas can be described by the following equation

$$\frac{d\mathbf{V}}{dt} + \frac{1}{\rho} \nabla p = 0 \quad (13)$$

where the vector $\mathbf{V}$ denotes the gas velocity. And it can be shown that $d\mathbf{V}/dt$ is given as

$$\frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = \frac{\partial \mathbf{V}}{\partial t} + \nabla \left(\frac{1}{2} \mathbf{V}^2 \right) - \mathbf{V} \times (\nabla \times \mathbf{V}) \quad (14)$$

Since the gas flow under consideration is irrotational i.e. $\text{curl} \mathbf{V} = 0$, a velocity potential ϕ exists [26] as follow

$$\mathbf{V} = -\nabla \phi \quad (15)$$

Substituting Eqs. (3), (14) and (15) into Eq. (13) gives

$$\nabla \frac{\partial \phi}{\partial t} - \nabla \left(\frac{1}{2} \mathbf{V}^2 \right) = \mathcal{R} \nabla T + \frac{\mathcal{R} T}{\rho} \nabla \rho \quad (16)$$

The equation of mass conservation can be shown

$$\nabla \cdot (\rho \mathbf{V}) + \frac{\partial \rho}{\partial t} = 0 \quad (17)$$

Substituting Eqs. (9), (10) and (15) into Eq. (17) yields

$$\frac{d}{dt} \{ \ln(1 + \bar{M}_a \chi_1) \} = \nabla^2 \phi \quad (18)$$

Use of Taylor series expansion and neglecting second and high order terms leads to

$$\frac{\partial (\bar{M}_a \chi_1)}{\partial t} = \nabla^2 \phi \quad (19)$$

¹ When $Q(T, t) = 0$ (i.e. no heat source), Eq. (11) can be simplified to $(1 + \bar{M}_a \tau_1) = (1 + \bar{M}_a \chi_1)^{\gamma-1}$, which describes the general adiabatic law in terms of $\bar{M}_a \chi_1$ and $\bar{M}_a \tau_1$.

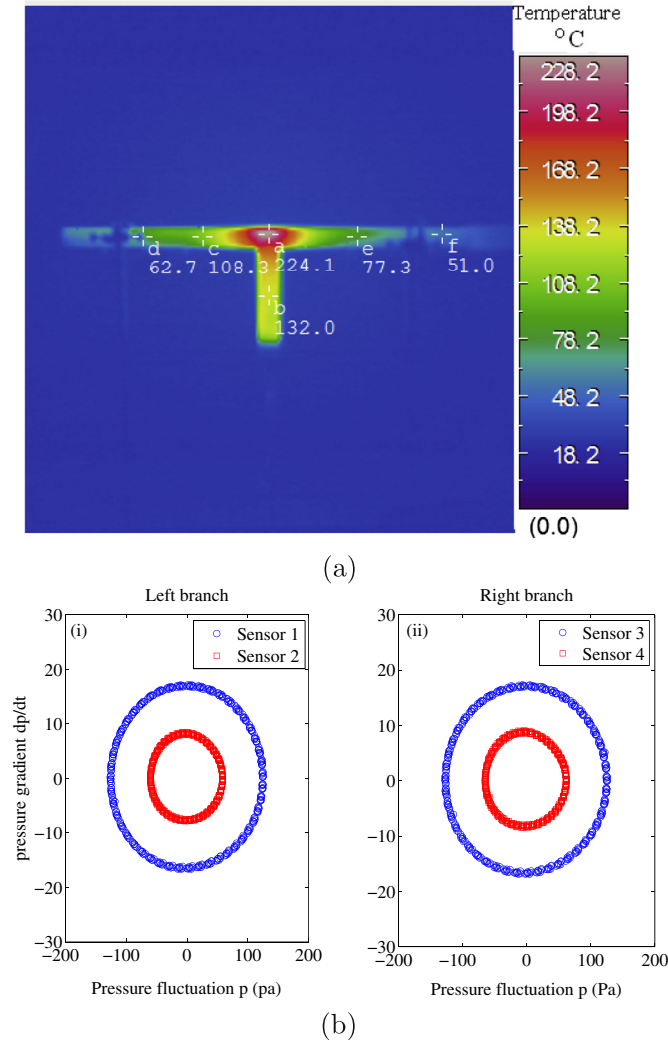


Fig. 6. (a) Temperature contour measurement of a bifurcating tube with $2\alpha = 180^\circ$ (b) phase diagram between $p(t)$ and $\dot{p}(t)$ along the left and right branches.

Similarly, we can linearize Eq. (16) by neglecting second order terms of $\bar{M}_a\tau_1$ and $\bar{M}_a\chi_1$ and such products as $\bar{M}_a\tau_1\nabla\bar{M}_a\chi_1$, and integrating it, then we obtain²

$$\frac{\partial\phi}{\partial t} = \mathcal{R}\bar{T}(\bar{M}_a\tau_1 + \bar{M}_a\chi_1) \quad (20)$$

Differentiating Eq. (20) with respect to t and replacing $\partial(\bar{M}_a\tau_1)/\partial t$ and $\partial(\bar{M}_a\chi_1)/\partial t$ with Eqs. (12) and (19) respectively gives

$$\frac{\partial^2\phi}{\partial t^2} - \bar{c}^2\nabla^2\phi = (\gamma - 1)\frac{\partial}{\partial t}Q(T, t) \quad (21)$$

where the speed of sound $\bar{c}^2 = \gamma\mathcal{R}\bar{T}$. When there is no thermal energy transferred into the system (i.e. $Q(T, t) = 0$), Eq. (21) reduces to the well-known acoustic wave equation

$$\frac{\partial^2\phi}{\partial t^2} - \bar{c}^2\nabla^2\phi = 0 \quad (22)$$

However, when there is thermal energy input (i.e. $\partial Q(T, t)/\partial t \neq 0$), Eq. (21) can be reformulated as

$$\frac{\partial^2\phi}{\partial t^2} - \bar{c}^2\nabla^2\phi = (\gamma - 1)\frac{\partial}{\partial t}Q(T) = (\gamma - 1)\frac{\partial}{\partial t}[\bar{Q}(T, t) + Q'(T, t)] \quad (23)$$

Eq. (23) describes the heat-driven acoustic wave equation. Further analysis on the right-hand-side of Eq. (23) shows that the unsteady heat addition $Q'(T, t)$ plays a dominant role in producing acoustic waves, while the mean part $\bar{Q}(T)$ does not make contribution. $\bar{Q}(T)$ is convected away by using heat exchangers as typically found in the conventional standing-wave system [18]. However, if $\bar{Q}(T)$ is harvested by using thermo-electric power generation module, then the thermal energy harvesting is maximized. This is achieved in our experiment.

If the acoustic damping effect such as the boundary losses, vorticity and flame-holders induced drag is involved, Eq. (23) can be shown as,

$$\frac{\partial^2\phi}{\partial t^2} + \zeta\frac{\partial\phi}{\partial t} - \bar{c}^2\nabla^2\phi = (\gamma - 1)\frac{\partial}{\partial t}Q(T, t) \quad (24)$$

where ζ denotes the overall damping coefficient. Substituting Eq. (6) into (24) leads to

$$\frac{\partial^2\phi}{\partial t^2} + \zeta\frac{\partial\phi}{\partial t} - \bar{c}^2\nabla^2\phi = \frac{(1 - \gamma)\mathcal{H}}{[C_3H_8]_{t=0}}\frac{\partial^2[C_3H_8]}{\partial t^2} \quad (25)$$

Since thermodynamic parameters such as pressure or density can be expressed in terms of velocity potential ϕ , Eq. (25) describes a general nonlinear heat-driven acoustic oscillation formulation by burning C_3H_8 . The heat-driven acoustic oscillation equation can also be applied to other fuels.

4.2. Thermoacoustic model

In order to illustrate the generation mechanism of acoustic oscillations by unsteady heat input and to qualitatively compare with experimental measurements, numerical simulations are conducted. A bifurcating tube with a localized heat source confined is simulated, corresponding to the experimental setup as shown in Fig. 9. The momentum and energy equations across the heat source, when linearized with respect to the perturbation quantities, can be written as

$$\frac{1}{\bar{c}^2}\frac{\partial p'}{\partial t} + \bar{\rho}\frac{\partial u'}{\partial x} = \frac{(\gamma - 1)}{\bar{c}^2}Q'(x_f, t) \quad (26)$$

$$\bar{\rho}\frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} = 0 \quad (27)$$

By differentiating Eq. (26) with respect to time t , Eq. (27) with respect to x and subtracting, an inhomogeneous heat-driven wave equation is obtained namely,

$$\frac{1}{\bar{c}^2}\frac{\partial^2 p'}{\partial t^2} + \frac{\zeta}{\bar{c}^2}\frac{\partial p'}{\partial t} - \frac{\partial^2 p'}{\partial x^2} = \frac{\gamma - 1}{\bar{c}^2}\frac{\partial}{\partial t}(Q'(t)\delta(x - x_f)) \quad (28)$$

where $\delta(x)$ is dirac delta function describing the localized heat source.

Expanding the pressure perturbation as Galerkin series [4,27] gives,

$$p'(x, t) = \sum_{m=1}^N \frac{-1}{\kappa_m\omega_m} \dot{\eta}_m(t)\psi_m(x) \quad (29)$$

where κ_m is a coefficient. Its physical meaning and derivation are shown later. N denotes the mode number. Overdot denotes the time

² Note that any additional term which depends on the time and not on the space coordinates is neglected from ϕ .

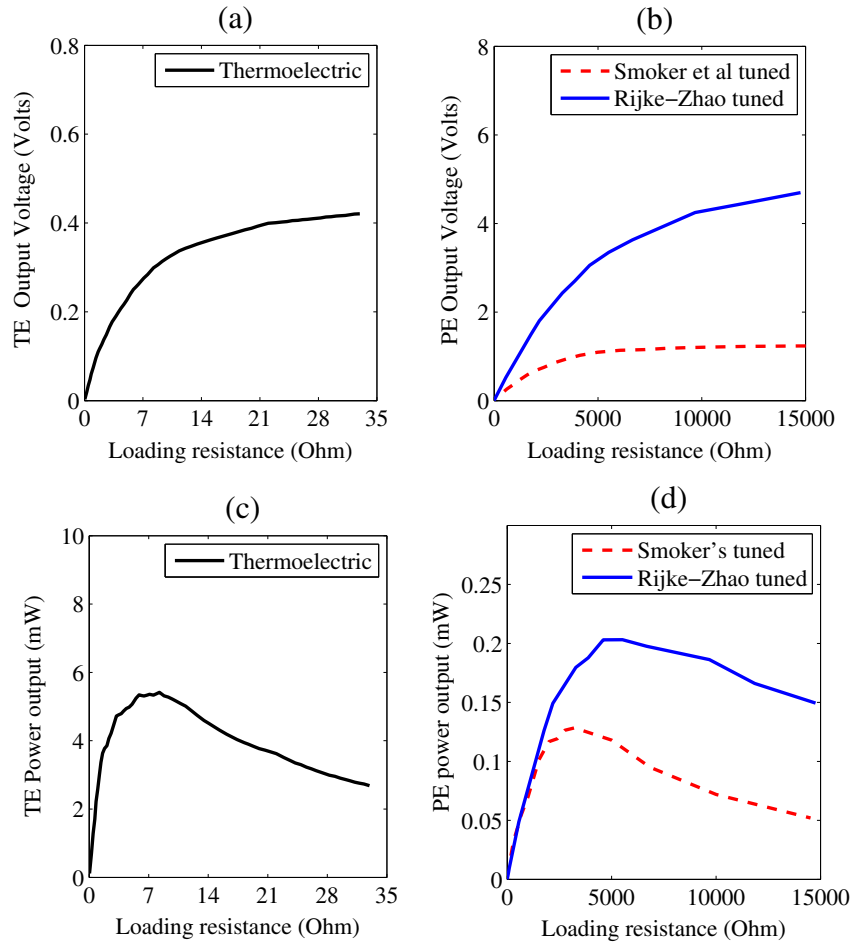


Fig. 7. Measured voltage and power output from the thermo- and piezo-electric generators with varying resistance R_L in Ω .

derivative. The function $\psi_m(x)$ are the eigen-solutions of the homogeneous wave equation, $\partial^2 p' / \partial t^2 - \bar{c}^2 \partial^2 p' / \partial x^2 = 0$.

The presence of the bifurcating branches at x_d , and the heat source resulting in the mean temperature undergoing a jump from the region $0 \leq x < x_f$ to $x_f < x \leq x_d$ give rise to 4 different flow regions. Substituting Eq. (29) into the homogeneous wave equation gives

$$\frac{\omega_m^2}{\bar{c}_i^2} \psi_m + \frac{d^2 \psi_m}{dx^2} = 0 \quad (30)$$

where $\bar{c}_i$ denotes the sound speed at i th region. It is assumed that there is no mean temperature jump from the region $x_f \leq x < x_d$ to $x_d < x \leq x_{d4}$. Thus $\bar{c}_4 = \bar{c}_2$ and $\bar{u}_4 = \bar{u}_2$. Mass and pressure continuity hold at $x = x_d$ as $\rho_2 u_2 S_2 + \rho_3 u_3 S_3 = \rho_4 u_4 S_4$ and $p_2(x_d, t) = p_3(x_d, t) = p_4(x_d, t)$. After linearizing them and neglecting the higher order terms, two jump conditions at x_d are given as

$$\left[\frac{d\psi_m}{dx} \right]_{x_d^-}^{x_d^+} = 0 \quad \text{and} \quad [\psi_m]_{x_d^-}^{x_d^+} = 0 \quad (31)$$

Applying the pressure continuity and velocity jump across the heat source and the boundary conditions at $x = 0$, $x = x_{d3}$ and $x = x_4$ [4,27], i.e. $\partial u'(x, t) / \partial x|_{x=0} = 0$, $\partial u'(x, t) / \partial x|_{x=x_{d3}} = 0$ and $\partial u'(x, t) / \partial x|_{x=x_{d4}} = 0$ leads to

$$\psi_m(x) = \begin{cases} \sin\left(\frac{\omega_m x}{\bar{c}_1}\right) & 0 \leq x < x_f \\ \hbar \left[\aleph \sin\left(\frac{\omega_m (x - x_d)}{\bar{c}_2}\right) + \cos\left(\frac{\omega_m (x - x_d)}{\bar{c}_2}\right) \right] & x_f < x \leq x_d \\ \frac{\hbar}{\sin\left(\frac{\omega_m (x_d - x_{d3})}{\bar{c}_3 \cos \alpha}\right)} \sin\left(\frac{\omega_m (x - x_{d3})}{\bar{c}_3 \cos \alpha}\right) & x_d < x \leq x_{d3} \\ \frac{\hbar}{\sin\left(\frac{\omega_m (x_d - x_{d4})}{\bar{c}_4 \cos \alpha}\right)} \sin\left(\frac{\omega_m (x - x_{d4})}{\bar{c}_4 \cos \alpha}\right) & x_d < x \leq x_{d4} \end{cases} \quad (32)$$

where $\hbar$ and $\aleph$ are given as

$$\aleph = \frac{\bar{c}_2 S_4 \cot(\omega_m (x_d - x_{d4}) / \bar{c}_4 \cos \alpha)}{S_2 \bar{c}_4 \cos \alpha} - \frac{\bar{c}_2 S_3 \cot(\omega_m (x_d - x_{d3}) / \bar{c}_3 \cos \alpha)}{S_2 \bar{c}_3 \cos \alpha} \quad (33)$$

$$\hbar = \frac{\sin(\omega_m x_f / \bar{c}_1)}{\aleph \sin(\omega_m (x_f - x_d) / \bar{c}_2) + \cos(\omega_m (x_f - x_d) / \bar{c}_2)} \quad (34)$$

where S denotes the cross-sectional area of each branch. $0 < \alpha < 90^\circ$ is half of the angle between the two bifurcating branches.

$\psi_m(x)$ satisfies the condition of orthogonality, i.e.

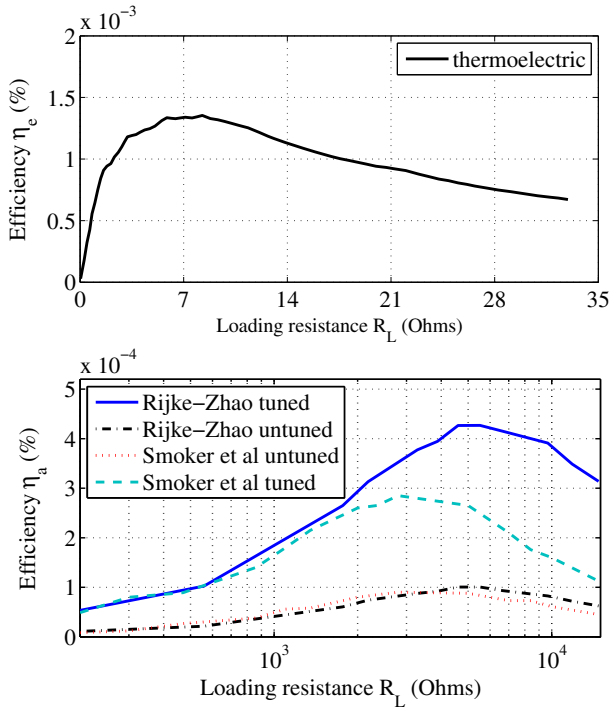


Fig. 8. Measured overall energy conversion efficiency from the present thermoacoustic system with the thermoelectric generator and tuned piezoelectric diaphragm. Note that the ‘tuned’ and ‘untuned’ denote the aluminum disk ‘attached’ and ‘unattached’ respectively.

$$\int_0^{x_d} \psi_m(x) \psi_n(x) dx + \int_{x_d}^{x_{d4}} \psi_m(x) \psi_n(x) dx = 0 \quad \text{for } m \neq n \quad (35)$$

Further analysis illustrates that the eigenmode frequency ω_m is satisfied with

$$\bar{\rho}_1 \bar{c}_1 \tan\left(\frac{\omega_m x_f}{\bar{c}_1}\right) = \frac{\kappa \sin\left(\frac{\omega_m (x_f - x_d)}{\bar{c}_2}\right) + \cos\left(\frac{\omega_m (x_f - x_d)}{\bar{c}_2}\right)}{\kappa \cos\left(\frac{\omega_m (x_f - x_d)}{\bar{c}_2}\right) - \sin\left(\frac{\omega_m (x_f - x_d)}{\bar{c}_2}\right)} \quad (36)$$

Substitution for $p'(x, t)$ from Eq. (29) into Eq. (27) lead to

$$\sum_{m=1}^N \left(\frac{d\eta_m^2}{dt^2} + \omega_m^2 \eta_m \right) \psi_m(x) = (\gamma - 1) \frac{\partial Q'}{\partial t} \delta(x - x_f) \quad (37)$$

After multiplication by $\psi_m(x)$ and integration with respect to x , the orthogonality condition of Eq. (35) shows that Eq. (37) can be simplified to

$$\frac{d\eta_m^2}{dt^2} + \omega_m^2 \eta_m = \frac{(\gamma - 1)}{\kappa_m^2} \int_0^{x_{d4}} \frac{\partial Q'}{\partial t} \delta(x - x_f) \psi_m(x) dx \quad (38)$$

where κ_m is given as

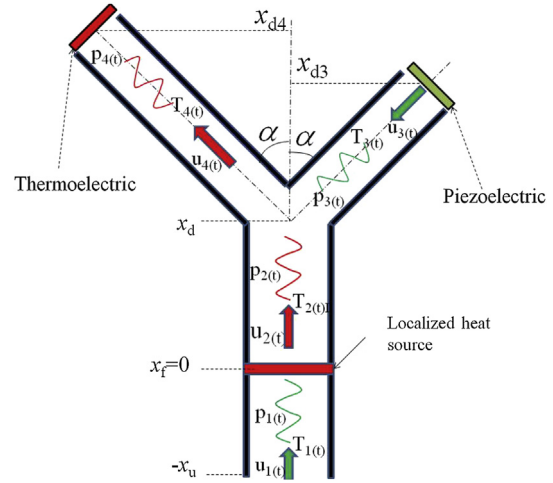


Fig. 9. Schematic of a dual-temperature thermoacoustic system with the thermoelectric and piezoelectric power generator attached.

$$\begin{aligned} \kappa_m^2 &= \int_0^{x_d} \psi_m^2(x) dx + \int_{x_d}^{x_{d4}} \psi_m^2(x) dx \\ &= \frac{1}{2} \left(x_f + (x_d - x_f) \left[\frac{h^2 \kappa^2 + h^2}{\sin^2(\omega_m (x_d - x_{d4})/\bar{c}_4 \cos \alpha)} \right] \right. \\ &\quad \left. + (x_{d4} - x_d) \frac{h^2}{\sin^2(\omega_m (x_d - x_{d4})/\bar{c}_4 \cos \alpha)} \right) \end{aligned} \quad (39)$$

Substituting Eq. (32) in Eq. (29) yields

$$p'(z, t) = \begin{cases} \sum_{m=1}^N \frac{-\dot{\eta}_m(t)}{\kappa_m \omega_m} \sin\left(\frac{\omega_m x}{\bar{c}_1}\right) & 0 \leq x < x_f \\ \sum_{m=1}^N \frac{-\dot{\eta}_m(t)}{\kappa_m \omega_m} h \left[\kappa \sin\left(\frac{\omega_m (x - x_d)}{\bar{c}_2}\right) + \cos\left(\frac{\omega_m (x - x_d)}{\bar{c}_2}\right) \right] & x_f < x \leq x_d \\ \sum_{m=1}^N \frac{-\dot{\eta}_m(t)}{\kappa_m \omega_m} h \sin\left(\frac{\omega_m (x - x_{d3})}{\bar{c}_3 \cos \alpha}\right) / \sin\left(\frac{\omega_m (x_d - x_{d3})}{\bar{c}_3 \cos \alpha}\right) & x_d < x \leq x_{d3} \\ \sum_{m=1}^N \frac{-\dot{\eta}_m(t)}{\kappa_m \omega_m} h \sin\left(\frac{\omega_m (x - x_{d4})}{\bar{c}_4 \cos \alpha}\right) / \sin\left(\frac{\omega_m (x_d - x_{d4})}{\bar{c}_4 \cos \alpha}\right) & x_d < x \leq x_{d4} \end{cases} \quad (40)$$

Substituting the Galerkin series expanded $p'(x, t)$ into Eq. (28), multiplying with $\psi_m(x)$ on both sides and integrating from $x = 0$ to $x = x_{d4}$ yields

$$\frac{\kappa_m}{\omega_m} \frac{d^2 \dot{\eta}}{dt^2} + \zeta \frac{\kappa_m}{\omega_m} \frac{d \dot{\eta}_m}{dt} + \kappa_m \omega_m \dot{\eta}_m = -(\gamma - 1) Q'(t) \sin\left(\frac{\omega_m x_f}{\bar{c}_1}\right) \quad (41)$$

The system Eq. (41) can be solved if $Q'(t)$ is known. In our experiment, the heat source is a premixed laminar flame. However, for simplicity, the localized heat source is numerically modeled as an electrical heater consisting of heated cylindrical hot wire in a perpendicular flow. Its unsteady heat release rate is described by using a modified form of King's Law [29] as:

$$Q'(t) = \frac{2L_w(T_w - T)}{S_1 \sqrt{3}} \sqrt{\pi \chi c_v \bar{\rho}} \frac{d_w}{2} \left(\left| \frac{\bar{u}}{3} + u'_f(t - \tau) \right|^{1/2} - \sqrt{\frac{\bar{u}}{3}} \right) \delta(x - x_f) \quad (42)$$

where L_w is the length of the wire, T_w is the wire temperature, T is the temperature of the surrounding air, χ is the heat conductivity of

the air, c_v is the specific heat of the air per unit mass at constant volume, $\bar{p}$ is the mean density of the surrounding air and d is the diameter of the wire and S_1 is the cross-sectional area of the tube. u'_f is the oncoming flow velocity and it can be estimated by using Eqs. (27) and (40) as

$$u'_f(t) = \sum_{m=1}^N \frac{\eta_m}{\bar{\rho}_1 \bar{c}_1 \kappa_m} \cos\left(\frac{\omega_m x_f}{\bar{c}}\right) \quad (43)$$

Now the time evolution of the heat-driven acoustic perturbations from specified initial conditions can be determined by numerically solving Eqs. (41) and (42) using a 4th Runge–Kutta method. In our calculations, the duct radius is 0.025 m, the angle between the two bifurcating branches with lengths of $x_{d3} = 0.783$ and $x_{d4} = 1.169$ m is $\alpha = 15^\circ$, the length of the mother tube is $x_d = 0.3$ m, and the heat source is placed at $x_f = 0.13$ m away from the bottom open end. The mean heat release rate $\bar{Q}(T)$ is set to 2.43×10^5 mW/m².

4.3. Rayleigh index and energy redistribution

Fig. 10(a) shows that the self-sustained thermoacoustic oscillation in terms of pressure is generated by introducing an initial impulse perturbation. The initial disturbance is quickly grown into limit cycle, as shown in Fig. 10(a). The resulting oscillations are at around 180 Hz with a sound pressure level 138 dB. Harmonics as an indicator of nonlinearity are also observed, as shown in Fig. 10(b). Comparing these numerical results with our experimental measurements, good agreement is obtained. Fig. 10(c) illustrates the variation of Rayleigh index Θ with time, which is generally used as a critical indicator of the thermoacoustic system stability [7,28]. It describes the energy exchange rate between unsteady heat release and the pressure oscillations. If $\Theta > 0$, the acoustical energy in the present system is increased and tends to drive the unstable process into a limit cycle. However, a negative Θ indicates a ‘destruction’ interaction between unsteady heat release and sound, and so the flow oscillations decay. Thus it would be interesting to check the

Rayleigh index Θ . Following the work [28], we define a normalized Rayleigh index $\Theta(t)$ as given as

$$\Theta(t) = \int_0^{x_d} \delta(x - x_f) dx \int_0^t \frac{p'(\omega, \xi) Q'(\omega, \xi) d\xi}{\sqrt{\sum_0^{2\pi/\omega} p'^2(\omega, \xi)} \sqrt{\int_0^{2\pi/\omega} Q'^2(\omega, \xi) d\xi}} d\xi \quad (44)$$

It can be seen that when initial flow disturbances grows into a limit cycle, unsteady heat release is in phase with the pressure fluctuation, i.e. $\Theta(\omega_1, t) > 0$. This phase information is consistent with Rayleigh's criterion [7], verifies our numerical model and explains why self-sustained thermoacoustic oscillations occur in the bifurcating tube.

To gain insight on the energy variation of the thermoacoustic modes, the energy redistribution between various modes are estimated. Fig. 11 shows the energy variation of the first 4 modes with time. The acoustical energy $\langle E(t) \rangle$ at ω_m is defined as

$$\langle E(\omega_m, t) \rangle = \frac{2\pi}{\omega_m} \int_t^{t+\frac{2\pi}{\omega_m}} \left[\int_0^{x_d} \left(\frac{p'^2(x, t)}{2\rho c^2} + \frac{\bar{\rho} u'^2(x, t)}{2} \right) dx dt + \int_{x_d}^{x_{d3}} \left(\frac{p'^2(x, t)}{2\rho c^2} + \frac{\bar{\rho} u'^2(x, t)}{2} \right) dx dt + \int_{x_d}^{x_{d4}} \left(\frac{p'^2(x, t)}{2\rho c^2} + \frac{\bar{\rho} u'^2(x, t)}{2} \right) dx dt \right] \quad (45)$$

The initial triggering disturbances with a small amplitude, such as impulse noise is set to ω_4 . However, the fundamental mode with the lowest frequency ω_1 is excited and grown into a limit cycle. As the initial disturbances are decaying, thermoacoustic modes at ω_2 and ω_3 also gain energy. The energy redistribution might result from the direct interaction between the two eigenmodes interact directly, or the interaction of various modes with the base flow.

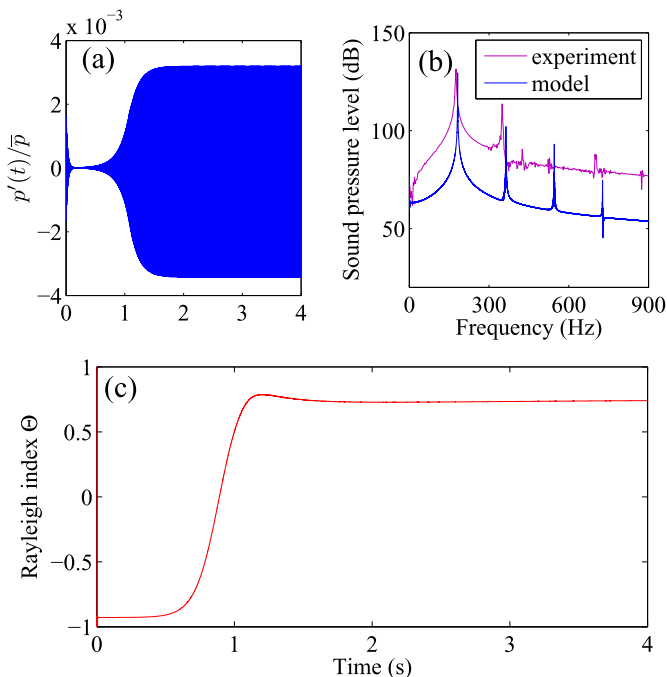


Fig. 10. Heat-driven acoustic oscillation in a bifurcating tube: (a) a limit cycle, (b) frequency spectra, (c) Rayleigh index $\Theta_1(\omega_1, t)$ variation with time.

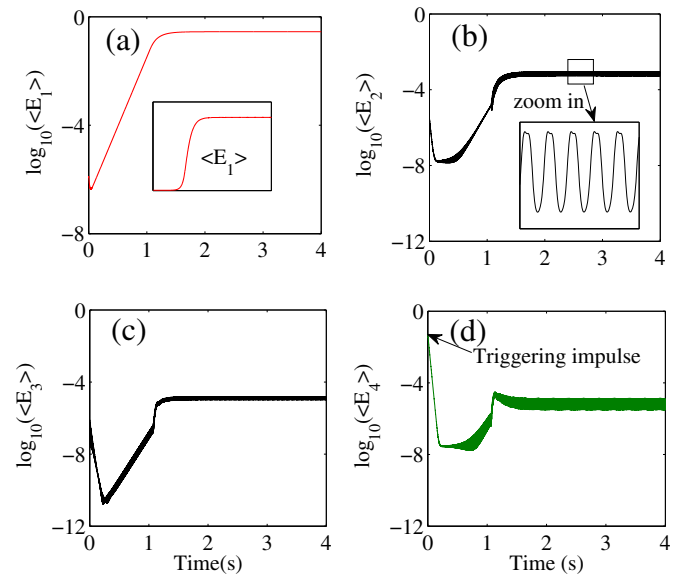


Fig. 11. Sound energy re-distribution: (a) variation of $\log_{10}(\langle E_1(\omega_1, t) \rangle)$ with time (b) variation of $\log_{10}(\langle E_2(\omega_2, t) \rangle)$ with time (c) variation of $\log_{10}(\langle E_3(\omega_3, t) \rangle)$ with time (d) variation of $\log_{10}(\langle E_4(\omega_4, t) \rangle)$ with time.

5. Conclusions

Thermodynamic measurement and analysis of dual-temperature thermoacoustic oscillations for energy harvesting application are conducted. A bifurcating tube is designed and tested to generate both 'hot' and 'cold' thermoacoustic waves. The acoustic fields in the bifurcating tube are monitored by implementing microphones, thermocouples, a hot wire anemometer and an infrared camera. The maximum sound pressure level is found at around 138 dB. Energy harvesting is achieved by using two different energy conversion processes via implementing both piezo- and thermo-electric power generation modules. This is different from the conventional standing-wave-piezo system [18], of which heat exchangers are involved. And only acoustical energy is harnessed, while the 'waste heat $\bar{Q}$ ' is dumped. It is found in our experiment that the total electric power output is 5.71 mW, of which approximately 5.5 mW is produced by the thermo-electric module. The power output is much larger than that harnessed from the standing-wave-piezo system.

To gain insight on the generation mechanism of the dual-temperature oscillations, a 'T-shaped' tube is tested. It is found that the angle between the bifurcating branches plays an important role. The angle effect is then considered in developing a nonlinear model to simulate our experiment. Thermodynamic laws are first used to derive a general formulation to illustrate how acoustic oscillations are produced by inputting thermal energy via burning C_3H_8 . The formulation is shown to be applicable to other type of fuels. It is also shown that unsteady heat release is an efficient sound source. And nonlinearity characterized by harmonics is associated with the thermoacoustic system. In comparison with our experimental measurement, qualitatively good agreement is obtained in terms of the oscillation frequency and sound pressure level. Rayleigh index Θ is then examined to gain insight on the interaction between unsteady heat release and oncoming flow disturbances. It is found that when $\Theta > 0$, sound energy converted from thermal energy keeps increasing and a limit cycle is eventually generated. Finally, the energy redistribution between various eigenmodes is examined. Sound energies of higher frequency modes are found to be transferred to the lower frequency ones. And lower frequency modes are easier to trigger.

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References

- [1] Swift GW. Thermoacoustic engines. *J Acoust Soc Am* 1988;84(4):1145–80.
- [2] Raun RL, Beckstead MW, Finlinson JC, Brooks KP. A review of Rijke tubes Rijke burners and related devices. *Prog Energy Combust Sci* 1993;19(5):313–64.
- [3] Zhao D, Morgans AS. Tuned passive control of combustion instabilities using multiple Helmholtz resonators. *J Sound Vib* 2009;320(2):744–57.
- [4] Zhao D. Transient growth of flow disturbances in triggering a Rijke tube combustion instability. *Combust Flame* 2012;159:2126–37.
- [5] Bisio G, Rubatto G, Sondhauss and Rijke oscillations—thermodynamic analysis possible applications and analogies. *Energy* 1999;24(2):117–31.
- [6] Li SH, Zhao D. Heat flux and acoustic power in a convection-driven T-shaped thermoacoustic system. *Energy Convers Manag* 2013;75(1):336–47.
- [7] Swift GW. Thermoacoustics – a unifying perspective for some engines and refrigerators. Melville, NY: Acoustical Society of America; 2002.
- [8] Rayleigh. A.K.A. (John William Strutt). The explanation of certain acoustical phenomena. *Nature (London)* 1878;18:319–21.
- [9] Cheng CH, Yang HS. Theoretical model for predicting thermodynamic behavior of thermal-lag stirling engine. *Energy* 2013;49(1):218–28.
- [10] Gardner DL, Swift GW. A cascade thermoacoustic engine. *J Acoust Soc Am* 2003;114(4):1905–19.
- [11] Yazaki T, Iwata A, Maekawa T, Tominaga A. Travelling wave thermoacoustic engine in a looped tube. *Phys Rev Lett* 1998;81(1):3128–31.
- [12] Markides CN, Smith TC. A dynamic model for the efficiency optimization of an oscillatory low grade heat engine. *Energy* 2011;36(12):6967–80.
- [13] Chen RL, Garrett SL. The solar/heat driven thermoacoustic engine. *J Acoust Soc Am* 1998;103(5):2841–2.
- [14] Moldenhauer S, Stark T, Holtmann C, Thess A. the pulse tube engine: a numerical and experimental approach on its design, performance, and operating conditions. *Energy* 2013;55:703–15.
- [15] Hu ZJ, Li ZY, Li Q. Evaluation of thermal efficiency and energy conversion of thermoacoustic stirling engines. *Energy Convers Manag* 2010;51:802–12.
- [16] Chun W, Oh SJ, Lee YJ, Lim SH, Surathu R, Chen K. Acoustic waves generated by a TA (thermoacoustic) laser pair. *Energy* 2012;45(1):541–5.
- [17] Rodriguez IA, Symko OG. Miniature traveling wave thermoacoustic engine. *J Acoust Soc Am* 2009;125(4):2562.
- [18] Smoker J, Nouh M, Aldrahem O, Baz A. Energy harvesting from a standing wave thermoacoustic-piezoelectric resonator. *J Appl Phys* 2012;111:104901.
- [19] Zhao D. Waste thermal energy harvesting from a convection-driven Rijke-Zhao thermo-acoustic-piezo system. *Energy Convers Manag* 2013;66:87–97.
- [20] Zhao D, Chew Y. Energy harvesting from a convection-driven Rijke-Zhao thermoacoustic engine. *J Appl Phys* 2013;112:87–97.
- [21] Linden PF. The fluid mechanics of natural ventilation. *J Fluid Mech* 1999;31:201–38.
- [22] Zhao D, Chow Z. Thermoacoustic instability of a laminar premixed flame in Rijke tube with a hydrodynamic region. *J Sound Vib* 2013;332:3419–37.
- [23] Turns SR. An introduction to combustion concepts and applications. New York, USA: McGraw-Hill; 2006. pp. 9–83.
- [24] Nicoli C, Pelce P. One dimensional model for the Rijke tube. *J Fluid Mech* 1989;202:83–96.
- [25] Jones H. The mechanics of vibrating flames in tubes. *Proc Royal Soc Lond Series A Math Phys Sci* 1977;353:459–73.
- [26] Dowling AP, Ffowcs-Williams JE. Sound and sources of sound. Chichester, West Sussex, England: Ellis Horwood Limited; 1983.
- [27] Dowling AP. The calculation of thermoacoustic oscillations. *J Sound Vib* 1995;180:557–81.
- [28] Pun W, Palm SL, Culick FEC. Combustion dynamics of an acoustically forced flame. *Combust Sci Tech* 2003;175:499–521.
- [29] Heckl MA. Nonlinear acoustic effects in the Rijke tube. *Acustica* 1990;72:63–71.